# Determining the Speed of Sound in Propane with a Rubens' Tube 

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## 1 The Big Idea

The idea of this project is to create a visual representation of how sound waves act in a closed tube. I am going to accomplish this through the creation of a Rubens' Tube, essentially a standing wave of fire. I got the idea for this project by perusing the Internet last year and it really stuck with me. I just never got over the fact that we could create such a perfect representation of what is actually going on inside of the tube with something as eye-catching as fire. Through creating this machine I hope to gain a deeper knowledge of how sound in a closed tube works, as well as how the density and temperature of a material effect the speed of sound in that material.

## 2 Introduction

This project is interesting and relative to me because it provides such a good visual of closed tube harmonics, something I struggled with in freshman physics. Being a visual learner I had trouble with why it was $1 / 4$ of a wavelength that was the fundamental frequency and why each of the next fundamental frequencies was $1 / 2$ of a wavelength higher. With a Rubens' Tube, however, it is easily visible why these phenomena occur; we can see what is going on inside the tube with the sound waves in the form of a standing wave of fire above the pipe.

Another interesting aspect of this project is the ability to measure the speed of sound in a tube varying with the temperature. Although it is already known that the speed of sound increases with the temperature, it will be very interesting to make my own measurements in a medium other than air. Will the difference due to temperature make itself more

[^0]or less apparent in the denser propane? In air the speed of sound can be calculated from this equation: $\mathrm{c}_{\text {air }}=\left(331.3+\left(0.606^{\circ} \mathrm{C}^{-1 *} \mathrm{~T}\right)\right) \mathrm{m}^{*} \mathrm{~s}^{-1}[1]$, so the question is whether or not the coefficient for propane, and other denser gases, is larger or smaller than the one for air.

## 3 Theory

Sound is the compression and decompression of air, or whatever medium it is traveling through. When the sound is played through a tube with holes at the top the resulting compressions and decompressions cause the gas to flow through the different holes in the top of the pipe at different pressures, and thus different heights. This height difference is then made visible by the burning of gas. In order to fully understand what is going on, however, the equation of $v=f \lambda$ is required, along with its use in relation to the harmonic frequencies of closed pipes ( $\lambda=4 \mathrm{~L}$, fundamental frequency $=\mathrm{v} / 4 \mathrm{~L}$ ) and the speed of sound in whatever flammable medium the sound will be traveling through. The Rubens' Tube that I am creating is closed on one end and open on the other, although it looks closed on both. One end is obviously closed, being sealed off by metal, but the other is closed by a rubber membrane, which can oscillate back and forth, thus essentially keeping that end open.

The $1^{\text {st }}$ harmonic, or fundamental frequency, is the note that has a wavelength four times as long as the pipe. This is because $1 / 4$ of its wavelength is exactly as long as the pipe, meaning that this particular frequency resonates with the pipe. This means that it will produce a much louder sound than the other frequencies. For closed pipes, this effect occurs whenever there is an anti-node at the open end of the pipe and a node at the closed end. We need an anti-node at the open end of the pipe because the air is free to vibrate up and down, and there must be a node at the bottom of the pipe because the air cannot vibrate up and down-thus a $1 / 4$ wavelength is the first possibility for the escaping sound to be the loudest.

These frequencies, with a node at the closed end and an anti-node at the open end, are called harmonics and have the possibility to occur
every time another $1 / 2$ of a wavelength is fit inside the pipe before the wave escapes the tube (because the distance between adjacent antinodes on a standing wave pattern is equivalent to $1 / 2$ of a wavelength [2]). In closed pipes we only get the odd harmonics because only one side of the pipe is changing-the other is consistently the same node, as you can see in Figure 1.


Figure 1: Standing waves in a half-closed pipe
This combined with the knowledge that the fundamental wavelength is simply four times the pipe's length allows us to calculate all of the harmonics of a closed pipe. For example, in a pipe with a length of four feet the fundamental frequency in propane at 70 degrees Fahrenheit (if only one end is closed) can be calculated by the equation $\mathrm{v}=\mathrm{f} \lambda .771 .47$ $\mathrm{ft} / \mathrm{s}=\mathrm{v}$ and $4^{\star} 4 \mathrm{ft}=\lambda$ (four times the length of the tube), thus giving us 48.22 Hz as our fundamental frequency. Then using our knowledge of harmonics we can calculate the $3^{\text {rd }}$ and $5^{\text {th }}$ harmonics. The $3^{\text {rd }}$ harmonic will occur when the pipe is $3 / 4$ the length of the wave, so our equation is now $771.47 \mathrm{ft} / \mathrm{s} /\left(4 / 3^{*} 4 \mathrm{ft}\right)=\mathrm{f}$, giving us 144.65 Hz . The $5^{\text {th }}$ harmonic occurs when the pipe is $5 / 4$ the length of the wave, so our equation then becomes $771.47 \mathrm{ft} / \mathrm{s} /\left(4 / 5^{*} 4 \mathrm{ft}\right)=\mathrm{f}$, giving us 241.08 Hz .

## 4 Results

The goal of this experiment was to calculate the change in the speed of sound in propane with respect to temperature. The first thing I did in the experiment was to find the change of temperature within the Rubens' Tube with respect to time (see Figure 2).


Figure 2: Temperature vs. time graph
Then, to make sense of this data, I calibrated the thermocouple I used to get the temperature in figure 2 against a thermometer known to be accurate. To calibrate it I took temperature readings from both instruments in ice water $\left(0^{\circ} \mathrm{C}\right)$, room temperature water $\left(\approx 20^{\circ} \mathrm{C}\right)$ and boiling water ( $100^{\circ} \mathrm{C}$ ) (see Figure 3).


Figure 3: Temperature with thermocouple vs. temperature with thermometer in ice water, in room-temperature water and in boiling water

I then took these three points to make a line that would calibrate the thermocouple's output reading to the real temperature as output by the thermometer (see Figure 4).



Figure 4: Thermocouple vs. thermometer calibration line

Then with this calibration line I was able to recalculate the temperatures that I got in figure 2 to be the real temperature inside the Rubens' Tube (see Figure 5). I also added the corresponding error bars from the thermocouple instruction manual, which says that the thermocouple is accurate within $\pm 2.2^{\circ} \mathrm{C}$ of its reading.


Figure 5: Calibrated thermocouple temperature with error bars
I then did an experiment where I discovered at which frequency the tube would display its $7^{\text {th }}$ harmonic, experimentally and theoretically, and changed the pitch of the speaker so that as the temperature rose, the $7^{\text {th }}$ harmonic remained visible. Combining this with my temperature readings during the test, $I$ got a $7^{\text {th }}$ harmonic vs. temperature graph (see Figure 6). I also included the $\pm 2.2^{\circ} \mathrm{C}$ error for the thermocouple and the statistical error for the frequency.

To calculate the theoretical frequency for the $7^{\text {th }}$ harmonic I used $\mathrm{v}=\mathrm{f} \lambda$, where $\mathrm{v}=$ the speed of sound in the material, $\mathrm{f}=$ the frequency and $\lambda=$ the wavelength. I used wolfram alpha, which could calculate the speed of sound in propane for certain temperatures, to get a line that gave me the theoretical speed of sound in propane (in $\mathrm{ft} / \mathrm{s}$ ). Then, using wave harmonics, I calculated that the wavelength of the $7^{\text {th }}$ harmonic for my 4 -foot long pipe should be 2.286 ft .


Figure 6: Temperature vs. frequency of the $7^{\text {th }}$ harmonic
Then, knowing the $7^{\text {th }}$ harmonic, we can deduce the speed of sound by dividing by the wavelength (length of the tube / (7/4)) (see Figures 7 \& 8). I thought this data would be linear, as it is for air, but it ended up looking more like a quadratic relationship (see Figure 8).


Figure 7: Temperature vs. the speed of sound in propane with a linear fit


Figure 8: Temperature vs. the speed of sound in propane with a quadratic fit
We can then compare this experimental data to what I got for the theoretical data (see Figure 9) and it is clearly linear.


Figure 9: Temperature vs. speed of sound theoretical data

This means one of two things: either my experimental data is a bit off and thus was not quite as linear as it should have been, or in reality the speed of sound in propane is not exactly linear. Instead it might be more quadratic, increasing when the temperature is higher and decreasing when the temperature is lower, down to a minimum speed. I think this could be quite plausible because the idea that the speed of sound changes exactly proportional to the temperature as the temperature drastically increases, which then changes the interaction between the molecules, sounds like an oversimplification of the world. It is rather like something that would work for most experiments that deal with moderate temperature change.

I also know that the sound I was using for this experiment was pure because I took a sample to check for any excess frequencies being output (see Figure 10). We know the sound is pure because the largest spike is at 1000 Hz .


Figure 10: Sound quality at 1000 Hz

In total, I know that my experiment was not tainted by impure sound nor did any other heating materials come into contact with the pipe during the test, but I still didn't get a linear result. Thus I believe that the speed of sound in propane changes with temperature almost linearly, instead following a very shallow quadratic of the formula (.1488 $\pm .01561 \mathrm{x}^{\wedge} 2-10.55 \pm 1.564+1065 \pm 38.16$ )

## 5 History

The Rubens' Tube was first invented in 1904 by Heinrich Rubens [4]. He created a tube that was four meters long and drilled 200 tiny holes in it, two centimeters apart, and then lit the holes on fire by pumping a flammable gas through the tube. Then, using the knowledge found earlier that century that acoustic standing waves would form in tubes (August Kundt) [3], Rubens played music through the tube and discovered the standing sine wave that is created on resonant frequencies. It has been repeated almost exactly the same way ever since.

## 6 Citations

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[^0]:    This paper was written for Dr. James Dann's Applied Science Research class in the spring of 2011.

